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Coupling of Waveguides Through Large Apertures

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Abstract—**A closed form expression for the equivalent network of a narrow transverse slot in the common broadwall between two waveguides is derived in terms of self-reaction and discontinuity in modal voltage. The coupling is expressed in terms of equivalent circuit parameter. A comparison between theoretical results with those obtained from the results of Levinson and Fredberg (for slot length $> 0.4a$, where a is the broad dimension of the waveguide), the results obtained from the formula of Sangster and the experimental results are presented.**

I. INTRODUCTION

IT HAS BEEN established by Levy [1] that the synthesis of a well-matched highly directive waveguide coupler reduces to the problem of designing a well-matched filter consisting of a cascade of shunt or series reactances. The application of the method demands an exact knowledge of the even and odd mode equivalent circuit parameters of the aperture in the common wall between two coupled waveguides. Further, the equivalent circuit representation makes possible the rapid determination of the waves emerging at all ports of a coupler using single as well as multiple apertures. In the latter case, it permits inclusion of the effects of reflection in between the apertures. The method of determination of equivalent circuit parameter and coupling therefrom, presented by Levy is limited to the apertures having small dimensions compared with wavelength. Marcuvitz [2] has given an expression for the susceptance presented by a transverse slot in the common broadwall between two waveguides, which is valid for the length of the slot equal to the broad dimension of the guide. Levinson and Fredberg [3] also have presented an analytical method for the determination of the equivalent circuit of apertures in the form of long narrow slots. Evaluation of the equivalent network by this method involves integrals of input admittances of coupled volumes, expressions for which have to be found from the solution of an integral equation [4]. No closed-form expression has been given for the aperture reactance as a function of slot length. For the particular case of two identical standard X-band rectangular waveguides, the curves for the reactance variation presented by Levinson and Fredberg [3] are valid for $2\ell/a > 0.4$.

The problem of electromagnetic coupling has been analyzed by a number of investigators [5]–[7]. The limitations of each of these methods have been discussed by Sangster [8] who applied the variational method for the analysis of coupling through narrow slots in the common wall between two rectangular waveguides. Sangster derived an expression for coupling between two waveguides coupled through slots in the common broadwall by employing an appropriate magnetic dyadic Green's function and using first-order trial function for the electric field distribution in the slot. He presented some computed and experimental results on coupling for a transverse slot in the common broadwall. The evaluation of coupling computed by the authors using Sangster's formula shows a deviation of 3 dB from the maximum coupling obtained from Levy's formula using an equivalent circuit approach, as well as with the experimental results by authors.

In the present paper a closed-form expression for the equivalent network parameter of a transverse slot in the common broadwall of a waveguide is derived in terms of self-reaction [9] and discontinuity in modal voltage [10]. The volume integral appearing in the expression for self-reaction is evaluated by replacing the slot by its equivalent cylindrical

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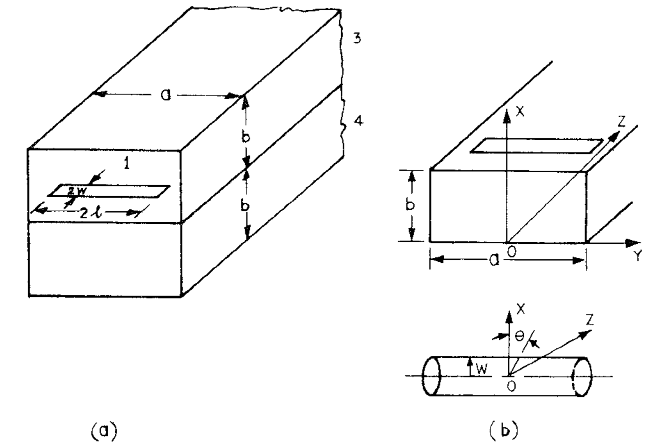

Fig. 1. (a) Transverse slot in the common broadwall of waveguides. (b) Coordinate geometry for rectangular waveguide and equivalent dipole.

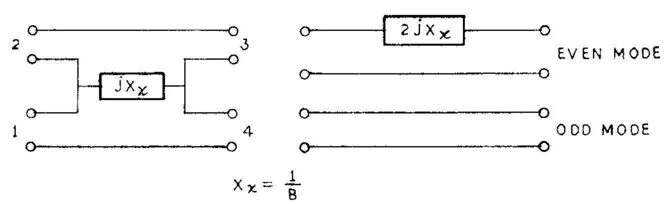

Fig. 2. Equivalent circuit of the slot.

magnetic dipole having a diameter equal to the slot width. The equivalent circuit is valid for any length of the transverse slot in the broadwall. From the knowledge of the equivalent circuit, the coupling due to the slot can be easily derived. The comparison between the computed results with those obtained from the results of Marcuvitz (for a full-length slot), Levinson and Fredberg (for $2\ell/a > 0.4$), the results obtained from the formula of Sangster, and the experimental results is presented.

II. Determination of Equivalent Circuit Parameter

Consider a transverse slot of length 2ℓ and width $2w$ in the common broadwall of two coupled rectangular waveguides as shown in Fig. 1(a). The equivalent circuit of such a slot is shown in Fig. 2. The susceptance appearing in the equivalent circuit is given by

$$B = \frac{\langle a,a \rangle}{\frac{1}{2}V^2} \tag{1}$$

where $\langle a,a \rangle$ is the self-reaction due to an assumed field distribution in the slot and V is the discontinuity in modal voltage for the dominant mode wave in the guide.

The electric field distribution $\bar{E}$ in the aperture plane of the slot is related to the equivalent magnetic current by the relation

$$\bar{J}_y^M = \bar{E} \times \bar{n} \tag{2}$$

where $\bar{n}$ is the unit normal to the aperture. Assuming that field distribution is of the form

$$\bar{E} = \bar{U}_z V_m \sin k(\ell - |y|) \tag{3}$$

the vector potential due to y directed current is given by [11]

$$A(x,y,z) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\varepsilon_n}{ab\gamma_{nm}} \cos\frac{n\pi x}{b} \sin\frac{m\pi}{a}\left(y + \frac{a}{2}\right) \cdot \int_S \left[\cos\frac{n\pi x'}{b} \sin\frac{m\pi}{a}\left(y' + \frac{a}{2}\right) \cdot e^{-\gamma_{nm}z} \int_{z'=-\infty}^{z'=z} \bar{J}_y^S(x',y',z') e^{\gamma_{nm}z'}\,dz' + e^{\gamma_{nm}z} \int_{z'=z}^{z=\infty} \bar{J}_y^S(x',y',z') e^{-\gamma_{nm}z'}\,dz'\right] ds \cdots \tag{4}$$

where the surface integration is performed over the cross section of the waveguide. $\varepsilon_n = 1$ when $n = 0$ and $\varepsilon_n = 2$ when $n = 1, 2, 3, \cdots$. The propagation constant

$$\gamma_{nm} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2}. \tag{4a}$$

It is known from the equivalence principle [9] that the slot can be replaced by an equivalent magnetic dipole. Fig. 1(b) shows such a dipole configuration with a coordinate system. The diameter of the cylinder is equal to the width of the slot.

The surface current on the cylindrical dipole is given by

$$\bar{J}_y^S = \bar{U}_y \frac{J_y^M}{2\pi w}. \tag{4b}$$

From the coordinate system shown in Fig. 1(b), the following relations are obtained:

$$x' = b + w \cos \phi \tag{5a}$$

$$z' = w \sin \phi. \tag{5b}$$

With the above substitutions, the integral in (4) gets considerably simplified. For a very narrow slot, the variation along the z' axis may be approximated by the delta function and $x' = b$. With these approximations it is found from (4)–(5b) that

$$A(x,y,z) = \sum_{n=0}^{\infty} \sum_{m=1,3,5}^{\infty} \frac{V_m \varepsilon_n}{2\pi wab\gamma_{nm}} \cdot \cos\frac{n\pi x}{b} \sin\frac{m\pi}{a}\left(y + \frac{a}{2}\right) \cdot \left[\frac{2 \sin\frac{m\pi}{2}\left(\cos\frac{m\pi\ell}{a} - \cos \ell\right)}{k\left\{1 - \left(\frac{m\lambda}{2a}\right)^2\right\}}\right] e^{-\gamma_{nm}z} \cdots \tag{6}$$

for $z > 0$.

The magnetic field H_y is related to the vector potential through the expression

$$H_y = \frac{1}{j\omega\mu}\left[k^2 A + \frac{\partial^2 A}{\partial y^2}\right]. \tag{7}$$

The self-reaction in (1) is given by

$$\langle a,a \rangle = -\int_v \bar{H}_y \cdot \bar{J}_y^S \, dv \tag{8}$$

where $\bar{H}_y$ is the magnetic field inside the waveguide due to the voltage on the slot. From (6)–(8) the expression for self-reaction reduces to

$$\langle a,a\rangle = \sum_{n=0}^{\infty} \sum_{m=1,3,5}^{\infty} \frac{4jV_m^2}{\pi^2 ab\omega\mu} \frac{\left(\cos \frac{m\pi\ell}{a} - \cos k\ell\right)^2}{\left\{1 - \left(\frac{m\lambda}{2a}\right)^2\right\}} \cdot \int_0^{2\pi} \cos\left(\frac{n\pi}{b} w \cos\phi\right) e^{-\gamma_{nm} w \sin\phi}\, d\phi. \quad (9)$$

Self-reaction $\langle a,a\rangle$ appearing in (9) is computed for $m = 1$ and $m > 1$ separately by the method suggested by Collin [12].

For $m = 1$

$$\langle a,a\rangle = \sum_{n=1}^{\infty} \frac{4jV_m^2}{\pi^2 ab\omega\mu} \frac{\left(\cos \frac{\pi\ell}{a} - \cos k\ell\right)^2}{1 - \left(\frac{\lambda}{2a}\right)^2} \cdot \int_0^{2\pi} \cos\left(\frac{n\pi}{b} w \cos\phi\right) e^{-\gamma_{n1} w \sin\phi}\, d\phi.$$

The cosine term appearing in the above integral is expressed as the sum of two exponentials and the summation and integration are interchanged. The series is summed by replacing $\gamma_{n1} \approx (n\pi/b)$ for large n. By introducing the appropriate correction term for the actual form of

$$\gamma_{n1} = \sqrt{\left(\frac{n\pi}{b}\right)^2 + \left(\frac{\pi}{a}\right)^2 - k^2}$$

it is found that the expression for the self-reaction can be approximated by

$$\langle a,a\rangle = \frac{4jV_m^2}{\pi^2 a\omega\mu} \cdot \frac{\left(\cos \frac{\pi\ell}{a} - \cos k\ell\right)^2}{1 - \left(\frac{\lambda}{2a}\right)^2} \cdot \left\{-2 \ln 2 \sin \frac{\pi w}{2b} + \frac{\beta^2 b^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^3}\right\} \quad (10a)$$

where

$$\beta^2 = k^2 - \left(\frac{\pi}{a}\right)^2.$$

For $m > 1$, Poisson's summation formula [12] is used to convert the sum into rapidly converging series as

$$\sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi}{b} w \cos\phi\right) e^{-\gamma_{n1} w \sin\phi}}{\gamma_{n1}} = \frac{b}{2\pi} \sum_{-\infty}^{\infty} 2k_0[k_m\{w^2 + (2nb)^2 + 4nbw \cos\phi\}^{1/2}]$$

where

$$k_m^2 = \left(\frac{m\pi}{a}\right)^2 - k^2.$$

Since the modified Bessel function of the second kind k_0 decays rapidly, the only significant terms are those for $n = 0$. Integration gives a factor of 2π only.

Hence for $m > 1$

$$\langle a,a\rangle = \sum_{m=3,5,7\cdots}^{\infty} \frac{4jV_m^2}{\pi^2 a\omega\mu} \frac{\left(\cos \frac{m\pi\ell}{a} - \cos k\ell\right)^2}{1 - \left(\frac{m\lambda}{2a}\right)^2} 2k_0(k_m w). \quad (10b)$$

Total reaction is obtained by addition of (10a) and (10b).

For calculating the required susceptance, the discontinuity in the modal voltage due to dominant mode wave is given by [10]

$$V = V_m \sqrt{\frac{2}{ab}} \left[\frac{2\left(\cos \frac{\pi\ell}{a} - \cos k\ell\right)}{k\left\{1 - \left(\frac{\lambda}{2a}\right)^2\right\}}\right]. \quad (11)$$

Hence, the required normalized susceptance can now be determined from (1), (10), and (11) as

$$\bar{B} = \frac{B}{y_0} = \frac{2kb}{\pi^2} \left[\left\{1 - \left(\frac{\lambda}{2a}\right)^2\right\} \cdot \left\{-2 \ln 2 \sin \frac{\pi w}{2b} + \frac{\beta^2 b^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^3}\right\} - \sum_{m=3,5,7\cdots}^{\infty} \frac{\left\{\cos \frac{m\pi\ell}{a} - \cos k\ell\right\}^2}{\left(\frac{m\lambda}{2a}\right)^2 - 1} \cdot 2k_0(k_m w) \left\{\frac{1 - \left(\frac{\lambda}{2a}\right)^2}{\cos \frac{\pi^1}{a} - \cos k^1}\right\}^2\right]. \quad (12)$$

The susceptance calculated from (12) is shown in Fig. 3 as a function of slot length for width of the slot $2w = 1$ mm and $\lambda = 3.2$ cm. Results given by Levinson and Fredberg (for $2\ell/a > 0.4$) are also shown for comparison. Close agreement is seen between the two results for large aperture lengths.

III. Determination of Coupling Between Two Waveguides

By using the even and odd mode equivalent circuit shown in Fig. 2, the relation between the coefficient of coupling and the equivalent circuit parameter is obtained as [1]

$$C_{\mathrm{dB}} = 20 \log \frac{1/\bar{B}}{2\,|(1 + j/\bar{B})|} + 8.686e^{-\alpha t} \quad (13)$$

where $\bar{B}$ is the normalized susceptance due to aperture. The second term in this expression refers to the correction in dB due to attenuation which occurs with given slot in a wall of finite wall thickness t. The attenuation constant

$$\alpha = \sqrt{\left(\frac{\pi}{2\ell}\right)^2 - k^2}$$

where 2ℓ is the length of slot and $k = 2\pi/\lambda$.

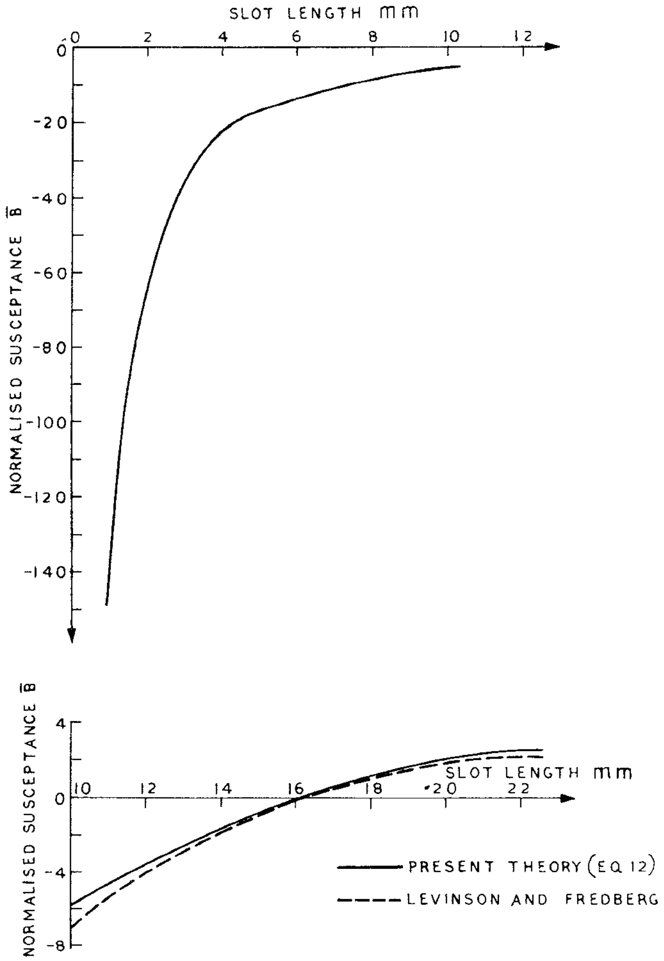

Fig. 3. Variation of normalized susceptance versus slot length.

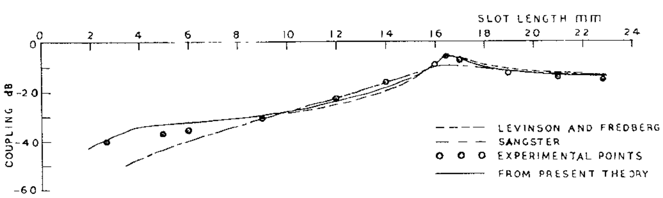

Fig. 4. Variation of coupling versus slot length.

Fig. 4 shows the theoretical results calculated from (12) and (13). For $2^l/a > 0.4$, the results compare quite closely with those calculated from the susceptance curve presented by Levinson and Fredberg [3]. Computed results from Sangster's formula [8] corrected for wall thickness t as mentioned above, are also shown in the same figure for comparison. For full length slot $2\ell = a$, the coupling calculated from the Marcuvitz's formula for the susceptance is 14.084 dB, which is close to that of our value of 14.804 dB. Coupling was also measured experimentally for a few slot lengths (slot width = 1 mm), covering the considerable range of the width of the broadwall (wall thickness $t = 1.58$ mm). The theoretical and experimental results are seen to be in good agreement.

IV. Conclusion

Theoretical results on coupling calculated using (13) are in fairly good agreement with experimental results. For $2\ell/a > 0.4$, the theoretical and experimental results on coupling are in good agreement with those obtained from the curve of susceptance variation presented by Levinson and Fredberg. It may, however, be pointed out that Levinson and Fredberg have not presented any expression from which the coupling for waveguides having different dimensions and operating in different frequency ranges can be evaluated. Further, the values of susceptances required for the calculation of coupling for transverse slots with $2\ell/a < 0.4$ cannot be obtained from his formulation. Coupling calculated from Sangster's formula shows a considerable deviation from the measured values of coupling in the neighborhood of maximum coupling as well as for low values of slot lengths. Further, Sangster's results are not convenient for the synthesis of multiaperture coupler by Levy's method.

The present analysis is quite general and can be used for determination of equivalent network and hence coupling for apertures in the common wall of any two arbitrary waveguides. Further, for two identical waveguides operating in any arbitrary frequency range, the results of present analysis can be employed for the synthesis of a well-matched highly directive coupler on the basis of the filter approach suggested by Levy.

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